It may be remarked that d is a mean proportional between a and m, l is a mean proportional between $3a$ and m, and l is the diagonal of a cube whose face-diagonal is s and edge d.

(6). To determine a point equidistant from all the vertices.

LK bisects MH at O.

In the rectangle of which GH and MN are opposite sides, GN bisects MH, therefore passes through O and is itself bisected at O.

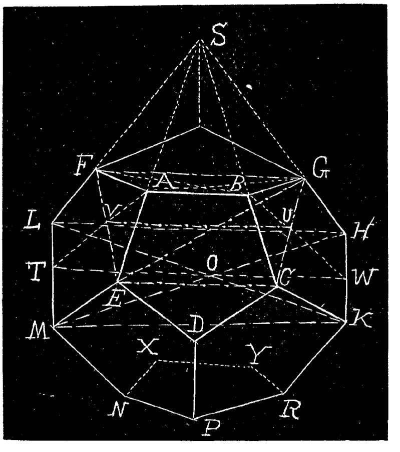
So with all the longest diagonals.

Hence O is equidistant from all the vertices.

Corollary 1. O is equidistant from all the edges.

Corollary 2. O is equidistant from all the faces.

(7). To compute the volume.



This could be done by conceiving the dodecahedron composed of twelve equal pyramids with O as their common vertex and the faces for bases. But another method will be adopted.

The dodecahedron is composed of a cube one of whose faces is $EFGC$ and six equal truncated triangular prisms, one of which has AB, FG, and EC for its lateral edges. A right section of this truncated prism is a triangle with base equal to EF, and altitude, the perpendicular from A to UV. Since $ABXY$ is a rectangle equal to $LMKH$, this perpendicular is one-half the difference between $AX(=LH)$ and an edge of the cube, and, therefore, equals $\frac{1}{2}a$.

Hence the area of the right section is $\frac{a^2}{8}[\sqrt{5}+1]$, and the volume of the truncated prism is $\frac{a^2}{8}[\sqrt{5}+1]\frac{a+a[\sqrt{5}+1]}{3}$.

The sum of the volumes of six such solids is $\frac{a^3}{4}[7+3\sqrt{5}]$.

Volume of cube$=\frac{a^3}{4}[8+4\sqrt{5}]$.

Volume of dodecahedron$=\frac{a^3}{4}[15+7\sqrt{5}]$.

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134. Proposed by J. C. GREGG, A. M., Superintendent of Schools, Brazil, Ind.

If $ABCD$ is a quadrilateral circumscribing a circle, show that the line joining the middle points of the diagonals AB, CD passes through the center of the circle.

Solution by HARRY S. VANDIVER, Bala, Penna.

We will use quadrilinear coördinates denoting the equation of the four

sides $\alpha=0$, $\beta=0$, $\gamma=0$, $\delta=0$, and the *lengths* of the corresponding sides by a, b, c, and d. Let radius of circle be r. Then the equation of the line joining the middle points of the diagonals is $a\alpha-b\beta+c\gamma-d\delta=0\ldots.(1)$ (Cf. Salmon's *Conics*, page 54, Ex. 5).

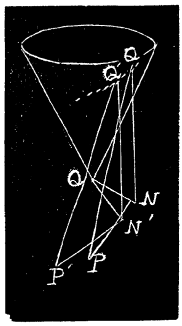
Putting $\alpha=\beta=\gamma=\delta=r$ we obtain $r(a-b+c-d)=0$, which is satisfied since $a+c=b+d$.

137. Proposed by J. W. YOUNG, Oliver Graduate Scholar in Mathematics, Cornell University, Ithaca, N. Y.

A right cone has its vertex in a horizontal plane, its axis being perpendicular to the plane. A string has one extremity attached to a point on the cone. The other extremity, P, of the string is kept in the plane, and the string is then wound around the cone, without being allowed to slip. Show that the spiral generated by P cuts all straight lines through the vertex at the same angle.

Solution by the PROPOSER.

Let P, P' be two points on the spiral; Q, Q' the corresponding points in the path of the string around the cone; N, N' the points where the perpendiculars from Q, Q' to the plane through the vertex O of the cone, cut the plane.



The right-angled triangles QNO, $Q'N'O$ have the angles QON and $Q'ON'$ equal; hence they are similar.

$\therefore \dfrac{QN}{ON}=\dfrac{Q'N'}{ON'}\ldots.(1).$

Again, since the string must not slip, it makes a constant angle with the plane.

$\therefore\ \triangle\, QNP$ is similar to $\triangle\, Q'N'P'$.

$\therefore \dfrac{PN}{QN}=\dfrac{P'N'}{Q'N'}\ldots.(2).$

From (1) and (2),

$\therefore \dfrac{PN}{ON}=\dfrac{P'N'}{ON'}\ldots.(3).$

But the triangles ONP and $ON'P'$ are right-angled at N and N' (PN, $P'N'$ being the projections to tangents to the circular cone). From (3),

$\therefore\ \triangle\, ONP$ is similar to $\triangle\, ON'P'$.

$\therefore\ \measuredangle OPN=\measuredangle OP'N'$.

Observing that PN and $P'N'$ are normals to the spiral, the last equation states that the normals make a constant angle with rays through O. Q. E. D.

AVERAGE AND PROBABILITY.

90. Proposed by WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.

During a heavy rain storm a circular pond is formed in a circular field. If a man undertakes to cross the field in the dark, what is the chance that he will walk into the pond? [From *Byerly's Integral Calculus*.]

I. Solution by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

Let O be the center of the circular field, and R its radius; C the center of